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Rubrique

Identification of source for the bidomain equation using topological gradient

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RÉSUMÉ. Une approche pour estimer les sources électriques dans le coeur à partir de mesures non invasives enregistrés sur la surface externe du thorax est proposé. L'approche est basé sur la méthode du gradient topologique. Cette méthode consiste à étudier le comportement d'une fonction coût au cours d'une perturbation dans le domaine. Nous montrons que notre approche proposée a effectivement été capable d'identifier le terme source et obtenir des résultats intéressants, et avec un coût de calcul particulièrement faible.

ABSTRACT. An approach for estimating electrical sources within the heart area from noninvasive measurements recorded on the outer surface of the thorax is proposed. The approach uses a topological gradient method. This method consists in studying the behavior of a cost function during a disturbance within the domain. We show that our proposed approach based on the topological gradient method has actually been able to perfectly adapt to the identification of the source term for obtaining very interesting results, and a particularly low computational cost.

MOTS-CLÉS : Le modèle bidomaine, Electrophysiologie cardiaque, Gradient topologique, problème adjoint, Analyse de sensibilité.

KEYWORDS : Bidomain model, Cardiac electrophsiology , Topological gradient, Adjoint problem, Analysis sensibility.

1 Introduction

Inverse problems are situations by which we seek to determine the causes of a phenomenon based on the observation of its effects. Some techniques, such as the regularization of ill-posed problems and the least squares method, were in place to help resolve such problems, whether linear or not. In this work our inverse problem is the identification of term source from measurements on a over some subset of the domain Ω, for example $\partial\Omega$ for the bidomain equation that describes the propagation of the electric wave in the heart (see [7, 11, 13]).

In this work, we focus on a recent method based on the topological gradient introduced by Sokolowski [12] and Masmoudi [10]. The topological gradient was originally used as part of the optimization shapes in solid mechanics, [8]. Then this approach has subsequently been applied to a large number of areas : in imaging, it was first used for the detection of contours [5], in image classification [2], inpainting [3] and segmentation [4].

A recent work [9, 6] have shown that the calculation of topological sensitivity associated with the cost function of the inverse problem provides good qualitative information on the location of obstacles identified. The topological sensitivity analysis giving the asymptotic behavior of the cost function when we perturb the domain, is expressed as a combination of direct solution and the adjoint state associated with the cost function, both defined in the absence of the obstacle. In this work we interest to the problem of the identification of source for the second equation of the bidomain model, ie we determine the term source of the equation that governed the extra cellular potential u_e. We consider the cardiac domain $\Omega_H \in \mathbb{R}^d$, $d = 2$ or 3, and the extra-cellular potential solution u_e of a system of partial differential equations defined in Ω_H as :

$$\begin{cases} -div((\sigma_i + \sigma_e)\nabla u_e) & = f & \text{in } Q \\ -div(\sigma_T \nabla u_T) & = 0 & \text{in } Q \\ \sigma_T \nabla u_T.n_T & = 0 & \text{on } \Sigma. \\ u_e & = u_T & \text{on } \Sigma, \\ \sigma_e \nabla u_e.n + \sigma_T \nabla u_T.n_T & = 0 & \text{on } \Sigma, \end{cases} \tag{1}$$

Where $f = div(\sigma_i \nabla V_m)$ is the source term to be identified. σ_i, σ_e and σ_T design respectively the intracellular, extracellular and thoracic conductivity tensors. And u_T is the thoracic potential. Furthermore define $Q = \Omega_H \times (0, T)$ and $\Sigma = \partial\Omega_H \times (0, T)$.
In order to identified the source from the data on a over some subset of the domain Ω_H , our approach is based on the least squares criterion :

$$j(\Omega_H) = \mathfrak{J}(u_\Omega)$$

Our goal is then to assess the sensitivity of the cost function $\mathfrak{J}$ when disrupts the study area by the insertion of a small subdomain ω_ϵ in the cardiac domain Ω_H. We assume that ω_ϵ has the form $\omega_\epsilon = x_0 + \epsilon\omega$, where $x_0 \in \Omega_H$, $\epsilon > 0$ and ω is a given, fixed and bounded domain of $\mathbb{R}^d$, containing the origin, whose boundary $\partial\omega$ is of C^1, ie to establish an expression of the form :

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) = \rho(\epsilon)g(x_0) + o(\rho(\epsilon))$$

where $\rho(\epsilon)$ is a function positive, tending to 0 as ϵ tends to 0, and the function $g(x_0)$ is called the topological gradient. When ϵ tends to 0 the cost function $\mathfrak{J}$ will be diminished. In order to establish this expression, it will be necessary to the asymptotic analysis of a problem zoomed added disruption to the original equation. We also introduce the solution p of the adjoint problem associated to the cost function $\mathfrak{J}$. A topological gradient calculation for unsteady problems (parabolic and hyperbolic) can be found in [1].

This paper is organized as follows we present in section 2 the general mathematical formulation of the forward problem and the adjoint method. Some examples of cost functionals are exhibited in Section 3. And the section 4 is devoted to the numerical results that validate the theoretical part.

2 The state problem

We assume that the cardiac domain to be located in a domain an open bounded subset denote Ω_H and Ω_T design the torso domain see figure (1). Our goal is to identify the source for the second equation of the bidomain equation

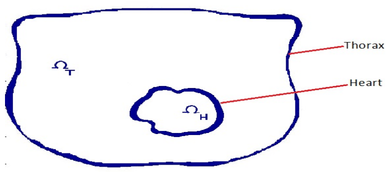

Figure 1 – The heart and torso domains

To the best of our knowledge, no work has been done in estimating the term source in cardiac electrophysiology using a approach based on the topological gradient. We consider here the simplified form of the second equation of the bidomain model given by :

$$\left\{ \begin{array}{rl} -div(\sigma \nabla u_\epsilon) & = f_\epsilon \;\text{ in }\; \Omega_H \times (0,T) \\ u_\epsilon & = 0 \;\text{ on }\; \partial\Omega_H, \end{array} \right. \tag{2}$$

where $u = u_e$ the extra-cellular potential and the $f_\epsilon = div(\sigma_i \nabla V_m)$
where

$$f_\epsilon = \left\{ \begin{array}{ll} f_1 & \text{on } \omega_\epsilon \\ f_0 & \text{on } \Omega_H \setminus \overline{\omega}_\epsilon \end{array} \right.$$

As noted in the introduction the topological gradient method consists in studying the variations of energy function from the perturbation of the domain.

2.1 Variational formulation

We define the the functional space by

$$V = \{ v \in H^1(\Omega_H) \quad , \quad v\backslash_{\partial\Omega_H} = 0 \}$$

and the bilinear form A_ϵ and the linear form l_ϵ as

$$A_\epsilon(u_\epsilon, v) = \int_{\Omega_H} \sigma \nabla u_\epsilon \nabla v \quad \forall v \in V$$

and

$$l_\epsilon(v) = \int_{\Omega_H} f_\epsilon v \quad \forall v \in V$$

Then the variational formulation of this problem reads such that

$$\int_{\Omega_H} \sigma \nabla u_\epsilon \nabla v = \int_{\Omega_H} f_\epsilon v$$

we deduce that u_ϵ is solution to

$$A_\epsilon(u_\epsilon, v) = l_\epsilon(v),$$

To determine the topological gradient we need to compute the adjoint solution of this problem. The is the aim of the next section.

2.2 Adjoint problem

We consider the direct solution u_ϵ verify $A_\epsilon(u, v) = l_\epsilon(v)$ and we define the lagrangian $L_\epsilon(u, v) = \mathfrak{J}(u) + A_\epsilon(u, v) - l_\epsilon(v)$, if u is solution of 2 we have

$$L_\epsilon(u, v) = \mathfrak{J}(u)$$

So

$$D_u L_\epsilon(u, v) = D_u \mathfrak{J}(u)$$

Then we define the abstract adjoint equation by

$$(D_u L_\epsilon, p) = 0$$

we have

$$(D_u \mathfrak{J}(u), p) + \int_{\Omega_H} \sigma \nabla p \nabla v = 0$$

So

$$\int_{\Omega_H} \sigma \nabla p \nabla v = -D_u \mathfrak{J}(u)$$

Finally the adjoint solution p associated of the cost function $\mathfrak{J}$ is given by

$$\left\{ \begin{array}{rll} -div(\sigma \nabla p) & = -D_u \mathfrak{J}(u) & \text{in } \ Q \\ p & = 0 & \text{on } \ \Sigma, \end{array} \right. \tag{3}$$

We remarque that the computation time and memory space required by the state adjoint method are largely reasonable. In the next section we will derive the variation of the cost function j with respect to the insertion of a small subdomain ω_ϵ in the fluid flow domain Ω_H. We begin our analysis by giving the main hypothesis 1, then the main result of this section is presented by Theorem 1. It concerns the topological asymptotic expansion of an cost function j.

2.3 Main result

The topological sensitivity theory provides a topological asymptotic expansion of j when ϵ tends to zero. It takes the general form

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) = \rho(\epsilon)g(x_0) + o(\rho(\epsilon))$$

Let us consider the following hypothesis :

hypothesis 1 *We assume That*
(i) $\mathfrak{J}$ is differentiable with respect to u, we denote $D\mathfrak{J}(u)$its derivative.
(ii) There exists a real number $\partial J(x_0)$ such that

$$\mathfrak{J}(u_\epsilon) - \mathfrak{J}(u_0) = D\mathfrak{J}(u_0)(u_\epsilon - u_0) + \epsilon^d|\omega_\epsilon|\partial J(x_0) + o(\epsilon^d)$$

(iii) $\|u_\epsilon - u\|^2_{L^2(\partial\Omega_H)} = o(\epsilon^d)$
(iv) $\|\nabla(u_\epsilon - u)\|^2_{L^2(\partial\Omega_H)} = o(\epsilon^d)$

The expression of the topological gradient for this problem is given by the following result :

Theorem 1 *Under the hypothesis above the cost function j has the following asymptotic expansion :*

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) = \epsilon^d|\omega_\epsilon|\partial J(x_0) - \epsilon^d|\omega_\epsilon|(f_1 - f_0)p(x_0)$$

In other words, the topological gradient at x_0 is :

$$g(x_0) = \partial J(x_0) - (f_1 - f_0)p(x_0)$$

where p is the adjoint solution.

Proof 1 *We always seek to minimize the function $\mathfrak{J}$ defined above. We consider the lagrangian*

$$L_\epsilon(u, v) = \mathfrak{J}(u) + A_\epsilon(u, v) - l_\epsilon(v)$$

u_ϵ is solution to 2 Then we have

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) = L_\epsilon(u_\epsilon, v)$$

So the first variation of the cost function with respect to ϵ is given by

$$\begin{aligned} j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) &= L_\epsilon(u_\epsilon, v) - L_0(u_0, v) \\ &= \mathfrak{J}(u_\epsilon) - \mathfrak{J}(u_0) + A_\epsilon(u_\epsilon, v) - A_0(u_0, v) - l_\epsilon(v) + l_0(v) \end{aligned}$$

Then from the definition of A_ϵ and l_ϵ we have :

$$A_\epsilon(u_\epsilon, v) - A_0(u_0, v) = \int_{\Omega_H} \sigma\nabla(u_\epsilon - u_0)\nabla v$$

$$l_\epsilon(v) - l_0(v) = \int_{\omega_\epsilon} (f_1 - f_0)v$$

Choosing $v = p$ the adjoint solution ie solution of 3

$$\int_{\Omega_H} \sigma \nabla(u_\epsilon - u_0) \nabla p = -D\mathfrak{J}(u_0)(u_\epsilon - u_0)$$

Then we have

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega) = \mathfrak{J}(u_\epsilon) - \mathfrak{J}(u_0) - DJ(u_0)(u_\epsilon - u_0) - \int_{\omega_\epsilon} (f_1 - f_0)p$$

From the hypothesis we have

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) = \epsilon^d |\omega_\epsilon| \partial J(x_0) - \epsilon^d |\omega_\epsilon| (f_1 - f_0) p(x_0)$$

So we have

$$j(\Omega_H \setminus \overline{\omega}_\epsilon) - j(\Omega_H) = \rho(\epsilon) g(x_0) + o(\rho(\epsilon))$$

where

$$g(x_0) = \partial J(x_0) - (f_1 - f_0) p(x_0)$$

where $\partial J(x_0)$ depend of the cost function. We will present in the previous section some examples of the cost function and the term $\partial J(x_0)$ associated.

3 Numerical results

We wish here to recover the source term with the help of the observation on the boundary. It is observed that the topological gradient method can identify the source in all positions and when the simulation time is small. Note that again, when we increase the time simulation the gradient topologique can not detect the terme source because it form as a wavefront. We consider a real life cardiac and thorax domain shown in the figures 2 and 3.In all of these figures we design by the point red by the stimulation point and the green point by the minimum of the topological gradient. The topological gradient algorithm is very easy to implement. In the different test we use the following algorithm :

- Resolve the forward solution of the problem 2.
- compute the adjoint solution of the problem 3.
- Compute the topological gradient g.
- Search for the minimum of the topological gradient.

3.1 Localized source :

We consider a two cost function $\mathfrak{J}_1(u) = \int_{\partial\Omega_H} |u - u_{obs}|^2 dx$ and $\mathfrak{J}_2(u) = \int_{\partial\Omega_H} |\nabla u - \nabla u_{obs}|^2 dx$ we have the following result :

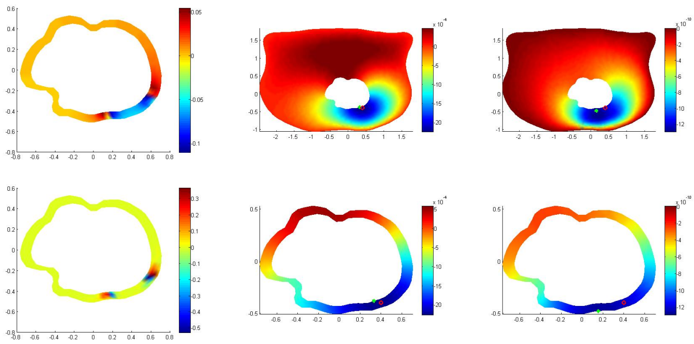

Figure 2 – Top (left) : the solution u_e at 4 ms, Bottom (left) : the source. Top (Middle) (respectively,Top (right)) The topological gradient for the cost function $\mathfrak{J}_1$ (respectively,$\mathfrak{J}_2$) in the heart thorax doamin. Bottom (Middle)(respectively,Bottom (right)) : The topological gradient for the cost function $\mathfrak{J}_1$ (respectively,$\mathfrak{J}_2$) in the heart doamin.

3.2 Distributed source

We consider the topological gradient when the time simulation is large,

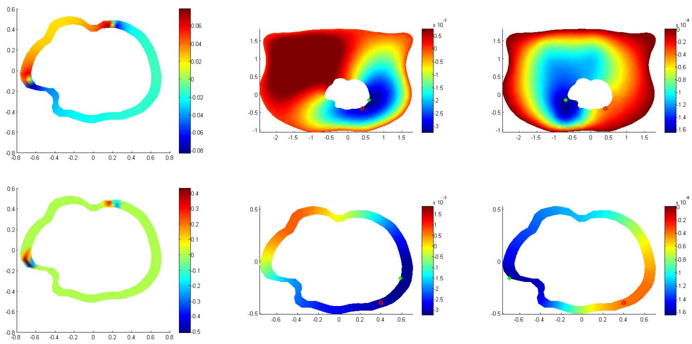

Figure 3 – Top (left) : the solution u_e at 20 ms, Bottom (left) : the source. Top (Middle) (respectively,Top (right)) The topological gradient for the cost function $\mathfrak{J}_1$ (respectively,$\mathfrak{J}_2$) in the heart thorax doamin. Bottom (Middle)(respectively,Bottom (right)) : The topological gradient for the cost function $\mathfrak{J}_1$ (respectively,$\mathfrak{J}_2$) in the heart doamin.

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